Notes 4.2, 4.6

Friday, October 26, 2012
6:39 AM

AGENDA:
- Quiz on 4.1
- Review Exams
- Notes 4.2, 4.6: Approximation Methods

4.1 #85. $V(0) = 25\ km/hr$ $V(13\ sec) = 80\ km/hr$

$$\frac{25\ \cancel{km}}{\cancel{hr}} \cdot \frac{1\ \cancel{hr}.}{60\ \cancel{min}.} \cdot \frac{1\ \cancel{min}}{60\ sec.} \cdot \frac{1000m}{1\ \cancel{km}} \Rightarrow 6.9\overline{4}\ m/sec$$

$$\frac{80km}{hr} \Rightarrow 22.\overline{2}\ m/sec$$

$$\boxed{V(0) = 6.9\overline{4}\ m/sec.} \quad \boxed{V(13) = 22.\overline{2}\ m/sec}$$

$$a(t) = K \Rightarrow V(t) = \int K\,dt = Kt + C$$

$$V(t) = Kt + C \Rightarrow 6.9\overline{4} = K(0) + C \rightarrow C = 6.9\overline{4}$$

$$\Rightarrow 22.\overline{2} = K(13) + 6.9\overline{4} \rightarrow K = \_\_\_\_$$

83. $V(t) = \dfrac{1}{\sqrt{t}}, \ t > 0$ $t = 1 \rightarrow x(1) = 4$

1) Find acc. function. $a(t) = V'(t) = \boxed{-\frac{1}{2} t^{-3/2}}$

2) Find pos. function: $x(t) = \int v(t)\,dt = \int t^{-1/2}\,dt$

$$x(t) = 2t^{1/2} + C$$

$$4 = 2(1)^{1/2} + C \rightarrow C = 2$$

$$\boxed{x(t) = 2\sqrt{t} + 2}$$

41. $\int (\tan^2 y + 1)\, dy$

$\int (\sec^2 x) dx = \tan x + C$

$* \quad 1 + \tan^2\theta = \sec^2\theta$

$$\int \sec^2 y\, dy = \boxed{\tan y + C}$$

43. $\int \frac{\cos x}{1 - \cos^2 x}\, dx$

$\sin^2\theta + \cos^2\theta = 1$

$$= \int \frac{\cos x}{\sin^2 x}\, dx = \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\, dx = \int \cot x \csc x\, dx$$

$$= \boxed{-\csc x + C}$$

Math 250 – Notes: Sect. 4.2/4.6 – Sigma Notation and AREA

I. Pre-Calculus Review: Sigma Notation

In Pre-Calculus, you study sequences and series. Recall that a *series* is just the **sum of a sequence.** The Greek letter sigma is used to denote "sum".

-example- Evaluate each sum.

a. $\sum_{i=1}^{5} 2i$ $= 2(1) + 2(2) + 2(3) + 2(4) + 2(5) = 2+4+6+8+10 = \boxed{30}$

b. $\sum_{n=1}^{4} \frac{(-1)^{n+1} n}{n+1}$ $= \frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} = \boxed{-\frac{13}{60}}$

c. $\sum_{k=0}^{3} (k+1)x^k$ $= 1x^0 + 2x^1 + 3x^2 + 4x^3$

$\boxed{1 + 2x + 3x^2 + 4x^3}$

Given a sum, we can write it in sigma notation if we can recognize a pattern.

-example- Write each sum using sigma notation.

a. $\frac{3}{1+1} + \frac{4}{1+2} + \frac{5}{1+3} + \ldots + \frac{12}{1+10}$ $= \sum_{i=1}^{10} \frac{i+2}{1+i}$

b. $\left(\frac{1}{2}\right)\left(1+\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)\left(1+\frac{2}{4}\right)^2 + \left(\frac{1}{2}\right)\left(1+\frac{3}{4}\right)^2 + \left(\frac{1}{2}\right)\left(1+\frac{4}{4}\right)^2$ $= \sum_{i=1}^{4} \frac{1}{2}\left(1+\frac{i}{4}\right)^2$

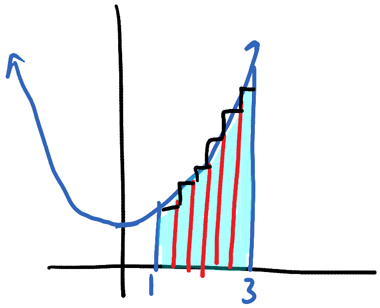
$= \frac{1}{2} \sum_{i=1}^{4} \left(1+\frac{i}{4}\right)^2$

Math 250 – Notes: Sect. 4.2/4.6 – Sigma Notation and AREA

II. Area

Consider this problem: Find the area of the region enclosed by the parabola, $y = 1 + x^2$, the vertical lines $x = 1$ and $x = 3$, and the x – axis.

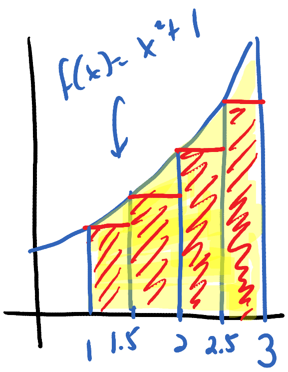
Picture:



Strategy: Since we know how to find the area of a RECTANGLE, we will divide the region into rectangular strips, and add the area.

Estimate 1: Use 4 *left endpoint rectangles* to approximate the area.

$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$



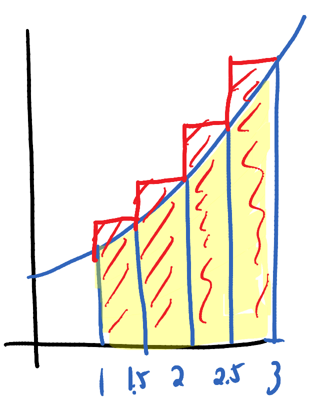
$A = b \cdot h = .5\,(f(x_i))$

$A = .5\left(f(1) + f(1.5) + f(2) + f(2.5)\right)$

$= .5\,(17.5) = 8.75$ (too low)

Sigma Notation: $\sum_{i=0}^{3} .5\,[f(x_i)]$

Estimate 2: Use 4 *right endpoint rectangles* to approximate the area.



$A = .5\left(f(1.5) + f(2) + f(2.5) + f(3)\right)$

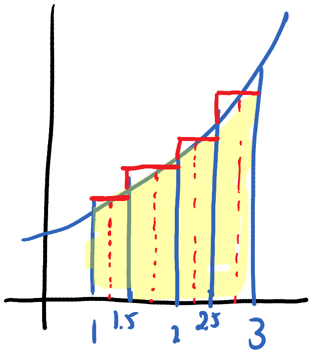
$= .5\,(25.5) = 12.75$ (too high)

Sigma Notation: $\sum_{i=1}^{4} .5\,[f(x_i)]$

Avg. of Left/Right: 10.75

Math 250 – Notes: Sect. 4.2/4.6 – Sigma Notation and AREA

Estimate 3: Use 4 *midpoint rectangles* to approximate the area.

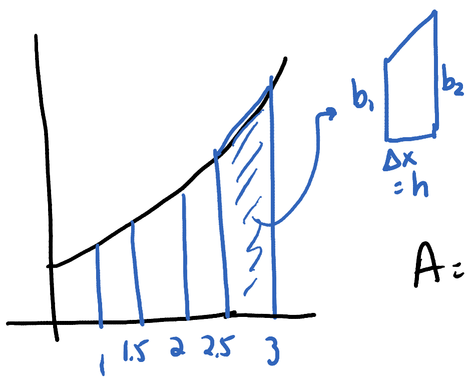


$$A = .5\left[f(1.25) + f(1.75) + f(2.25) + f(2.75)\right]$$

$$A = .5\,(21.25) = \boxed{10.625}$$

Sigma Notation: $\sum_{i=0}^{3} f\left(\frac{x_i + x_{i+1}}{2}\right)\Delta x$

Estimate 4: Use 4 *trapezoids* to approximate the area.



$$A = \tfrac{1}{2}h(b_1 + b_2)$$

$$A = \tfrac{1}{2}\left(\tfrac{1}{2}\right)\left[f(1) + 2f(1.5) + 2f(2) + 2f(2.5) + f(3)\right]$$

$$= \tfrac{1}{4}(\_\_\_\_) = \_\_\_\_\_\_$$

RECTANGLE APPROXIMATIONS are called *Riemann Sums*. The form of a Riemann Sum is: $\sum_{i=1}^{n} f(x_i)\Delta x_i$

To find the exact area, we need: $\lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i)\Delta x_i$

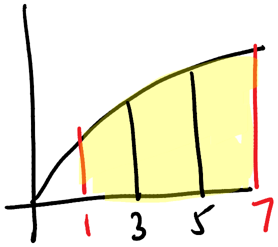
New notation for this limit: ***Definite Integral*** $\int_a^b f(x)\,dx$

3

Math 250 – Notes: Sect. 4.2/4.6 – Sigma Notation and AREA

Further Practice: Consider the function $f(x) = \sqrt{x}$ from $x = 1$ to $x = 7$.

a. Use 3 subintervals of equal length and *midpoint rectangles* to approximate the value of $\int_1^7 \sqrt{x}\,dx$.

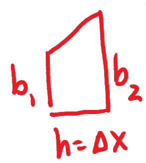


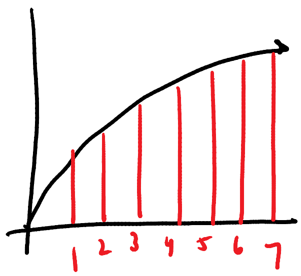
$$\int_1^7 \sqrt{x}\,dx \approx 2\left[f(2) + f(4) + f(6)\right]$$

$$= 2[\quad] = \underline{\qquad\qquad}$$

b. Use 6 subintervals of equal length and *trapezoids* to approximate the value of $\int_1^7 \sqrt{x}\,dx$. (Sect 4.6)

$A_{trap} = \frac{1}{2}h\,(b_1 + b_2)$





$$\Delta x = \frac{7-1}{6} = \frac{6}{6} = 1$$

$$\int_1^7 \sqrt{x}\,dx \approx \frac{1}{2}(1)\left[f(1) + 2f(2) + 2f(3) + 2f(4) + 2f(5) + 2f(6) + f(7)\right]$$

$$= \underline{\qquad\qquad}$$